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DYNAMICS OF SPRINGY SYSTEMS OF PACKAGING LINES FOR FOOD PRODUCTS

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Abstract: *The purpose of the study was to develop mathematical models and proposals for improving the technology and equipment for the supply of roll packaging materials. The analysis of features of systems for transportation of packing materials, dynamics of transient processes, research methods concerning movements of rolled materials, features of loadings in systems of cyclic action with springy elements are resulted in the article. It is shown that the modern theoretical basis of synthesis of technological machines on the basis of use of film rolled materials combines possibilities of the account of technological requirements, indicators of high productivity, neighborhood conditions, restriction of power parameters. Mathematical formalizations concerning the ratios of force factors in systems of transportation of loads of cyclic action at the levels of external force actions, generated friction forces, geometrical, kinematic and dynamic parameters were obtained. Mathematical models concerning runaway modes and estimation of stiffness of springy bonds, on the basis of film materials, methods of estimation of system parameters were offered. Synchronized combination of operations, the use of optimal laws of movement of working bodies, limitation of dynamic loads was carried to the positive directions of creating efficient technological machines and lines. It is proposed to determine the ratio of the run-out time of rolls of variable mass by the ratio of the squares of the initial and final radii. The solution of the problem of kinetic energy recovery in systems of periodic action is simultaneously connected with the restriction of the non-uniformity of the drive of machines.*

Keywords: *dynamics, springy connections, packaging lines, transient processes, mechanical systems.*

1. Introduction

Intensive innovations in food production technologies are accompanied by the introduction of the latest equipment in product packaging lines. This applies to packaging made of flexible, semi-rigid and rigid materials. An important component of modern packaging lines is that in many cases they contain raw materials and equipment for the manufacture of blanks, the work of which is synchronized with the subsequent operations of packaging, sealing, design and creation of group packaging. The list of modern flexible packaging materials satisfies the packaging

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industry of any society at the state level, reflecting the development of the economy, agricultural sector, social development. Packaging materials in combination with physico-chemical methods of food stabilization and packaging technologies provide regulatory requirements for warehousing, transportation, storage and sale of products. A promising direction in the development of modern food production is the concentration of production capacity and increasing the range of products in different types and sizes of packaging. The latter means the feasibility of using systems with universal flexible equipment for reconfiguration.

The dynamics of development of production of packaging materials and technological equipment is approaching the exponential law, which has the prospect of abrupt transformations rather than evolutionary changes. Expectations of important and fleeting changes are projected on the basis of the development of scientific base and empirical achievements in existing technologies, as well as on the basis of modern combination of mechanics and electronics, which are accompanied by the possibility for implementing high-precision laws in dynamics with load and energy costs. Such a dualism of the final result is possible on the basis of in-depth assessments in the course of transients and established modes of operation under continuous and discrete conditions. Hence, the issues of synthesis of packaging equipment are shifted in the areas of universality, reliability, accuracy of operation of working bodies, economy modes, economic efficiency.

However, the synthesis of modern high-performance machines is accompanied by the need to overcome the barrier of natural contradiction, on the other side of which there are dynamic loads. High productivity of the car means necessity to carry out the set movements in limited time with considerable accelerations and inertial loadings. The use of technological machines of discrete action leads to the need to overcome such contradictions inherent in the systems for the supply of roll packaging materials. The presence of rolls with significant masses in them leads to complications of the modes of stabilization of the movements of the film canvas and their tensions at distances between the rolls and the leading working bodies.

A feature of the providing systems is the use of friction drives in various forms. This applies to the mechanisms of rotation of the rolls, regulation of their runoff modes, the device of compensation and damping devices and devices for driving film canvas.

The inability to achieve synchronization in the modes of run-out of the leading masses with the working bodies and the driven masses of the rolls led to the use of compensation and damping devices, which divide the system into almost two dynamically independent parts. In connection with this division, there is a need for in-depth analysis of the supply systems of rolled packaging materials in order to stabilize the kinematic parameters in their double parts, the tension of the film canvas in the areas between compensation and shock working devices, improving synchronization modes and increasing friction systems.

The purpose of the study was to develop mathematical models and proposals for improving the technology and equipment for systems for the supply of roll packaging materials.

Based on the analysis of their construction, the following **research objectives** were formulated:

- to perform analysis and generalization, which relates to the features of the systems of movement of rolled packaging film materials;

- to develop a mathematical apparatus for describing transients in systems for moving film materials, separated by compensation and depreciation devices;
- to develop recommendations for limiting the dynamic components of film flow loads, the possibilities of kinetic energy recovery and limiting the unevenness of the stroke in the drives of the equipment.

The dynamics of transients in technological machines continues to be an important component, thanks to which scientists have information on loads, power supply, uniformity of machine systems. Of course, in-depth load forecasting applies to complex engineering structures, but in conditions of non-stationary external influences, it is advisable to supplement them with the volume of operational data [1]. The result of this approach was not only the prediction of the dynamic behavior of loads, but also an accurate assessment of their statistical characteristics with data on capacity.

The direction of achieving accuracy in the performance of technological operations involved the use of special measures at the level of dynamic compensation of the reactions of systems exposed to environmental vibrations and other influences [2].

The accuracy of the manufacture of special purpose parts required special monitoring of transients, which was realized due to the known relationship between power consumption and power performance. However, the manifestations of the present control devices and the effects of transmissions led to errors in determining the actual force. The calculation of a twisted model based on the effective power signal to compensate differences was proposed [3]. The creation of methods for the study of transients was being actively improved. The study of the mechanical system with hydrodynamic power transmission under monoharmonic loading, which was carried out by introducing a stationary calculation of an immeasurable mathematical model was an example [4]. For this purpose, a modified method of harmonic balance was used.

The effects of radial forces and machining were accuracy combined on the basis of the equivalent contact load model and Hertz's theory [5] led to estimation of errors in the manufacture of details.

Inertial loads corresponded to transients and their values had significant manifestations in high-speed machines. At high speeds and accelerations, the relationships between the subsystems became more complicated, which required the use of a new research method. It was shown [6] that at limited speeds and accelerations inertial forces had little effect on system performance. This means the expediency of their restrictions in certain ranges.

In the study [7] the evaluation of the effects of machine vibration isolation on the dynamic loads of bearings with the determination of the range of dynamic reactions and the possibilities of their limitations was made.

Dynamic prediction of the characteristics of a 5-DOF hybrid machine based on a scale model taking into account deformations is an example of the possibilities of analyzing systems taking into account changes in mass and stiffness [8].

The transition processes of start and idling of machines related to energy loads and energy costs within the interests and balances of generation and consumption systems [9, 10] proposed a procedure for limiting peak loads by implementing certain laws.

Energy saving technologies logically

extend to various areas of technological machines [11] and the stability of multi-machine power systems [12]. Increased energy losses were related to cyclic machines, which are largely represented in the packaging lines. Peculiarities of the course of transients in these cases did not allow to use the properties of dynamic systems based on the control effects of flywheels or flywheel masses [13]. However, the driven masses of technological carousel machines were commensurate with the reduced driving masses, which accordingly determined the potentials of their kinetic energy and the feasibility of organizing recuperative regimes.
In connection with this, the regulation of the movement of machines and the kinematics of movements of the working bodies in combination provided a double result in terms of limiting the dynamic loads and energy consumption.

2. Materials and methods

Methods of mathematical modeling of physical and mechanical processes of interactions of working bodies of the equipment with film material streams, kinematic and dynamic synthesis of elements of systems, mathematical and statistical analysis of results of experiments were used.
Methods of mathematical programming using universal engineering-mathematical complex MatLAB were used for carrying out mathematical calculations and results of experiments.

3. Results and discussion

3.1. The results of the study of the system for providing rolled packaging materials. Dynamics of driven masses of rolls

In technical generalization, it was considered that the devices for providing rolled packaging materials were represented by three groups [14]. These included groups of main, auxiliary and additional working bodies.
The main group was represented by roll holders, devices for moving materials and equipment for cutting and trimming material.
Auxiliary working bodies provided accumulation and amortization of materials, braking of rolls, braking and stopping of film streams and the direction of their movements.
Additional working bodies controlled the flows of materials and products, modes of unwinding of rolls, stocks of film in a roll, adjustment of kinematic parameters, automatic filling of the film and providing of the film from various rolls.
The working bodies of the main and auxiliary groups directly interacted with the rolls and film canvas, and the bodies of the additional group performed control, blocking, out-of-cycle, regulating and other similar operations [15, 16]. Their functional tasks were to control the operation of devices for providing packaging materials and other devices of packaging machines, which had malfunctions of the film supply system.
An important dynamic component of the supply system of rolled packaging materials was a roll holder with a roll of film material. The task entrusted to the roll holder was to ensure the appropriate positioning of the roll in the specified coordinates with the possibility of rotational movement in the mode of winding the film. The relatively significant mass of the roll holder and the roll in dynamic models of the system led to the fact that they were collectively attributed to the driven masses, which in transients determined a significant proportion of dynamic loads [17].
Obviously, the design of the roll holder was directly related to the weight of the

roll and to some extent with the mode of operation of the packaging line. On the one hand, this formulated the requirement to limit the weight of the roll holder, but on the other hand, the design must have ensured the reliability of the specified coordinates and position of the roll and the possibility of its rapid replacement.

The driving factor in the transition process was the tension of the film material, which depended on its load on the working body, its law of motion and the ability to develop and transmit force to the film canvas. Obviously, from the point of view of the interests of finding the optimal parameters of the components of the system, a positive result was quite achievable, but it corresponded to the stabilized values of the driving and driven masses and stiffness. If with respect to the driving mass and stiffness such a condition was acceptable, then with respect to the driven mass, its changes by an order of magnitude or more were predicted. The last conclusion was connected with change of weight of a roll and its diameter at the beginning and at the end of winding film from it. The maximum and minimum values of the moments of inertia of the combination of the roll holder and the roll meant the effects on the maximum and minimum values of the springy loads of the film canvas. In cyclically operating equipment, transient modes of acceleration and run-out accompanied the entire technological time of operation of the equipment and this was the reason for finding opportunities to stabilize the dynamic parameters. To confirm this position, we turned to the comparison of the moments of inertia of the driven mass in the range of changes in the radius of the roll from the initial $R_{(i)}$ to the final $R_{(f)}$ at $R_{(i)} > R_{(f)}$. Under such conditions we have:

$$I_{(i)} = \frac{m_{(i)} R_{(i)}^2}{2}; \quad I_{(f)} = \frac{m_{(f)} R_{(f)}^2}{2}; \qquad (1)$$

since not only radii but also masses will be variable $m_{(i)}$ and $m_{(f)}$ as the initial and final, respectively.

Let's display changes of masses through geometrical parameters and specific weight ρ of a film material:

$$m_{(i)} = \rho V_{(i)}; \quad m_{(f)} = \rho V_{(f)}, \qquad (2)$$

where $V_{(i)}$ and $V_{(f)}$ –the initial and final volumes of the roll, respectively, which were determined by the dependences:

$$V_{(i)} = \pi R_{(i)}^2 H; \quad V_{(f)} = \pi R_{(f)}^2 H \qquad (3)$$

The corresponding substitution give the value:

$$m_{(i)} = \rho \pi R_{(i)}^2 H; \quad m_{(f)} = \rho \pi R_{(f)}^2 H, \qquad (4)$$

where H – roll height.

Then,

$$I_{(i)} = 0.5 \pi R_{(i)}^4 H; \quad I_{(f)} = 0.5 \pi R_{(f)}^4 H \qquad (5)$$

Then the ratio $I_{(i)}/I_{(f)}$ is:

$$\frac{I_{(i)}}{I_{(f)}} = \frac{R_{(i)}^4}{R_{(f)}^4} \qquad (6)$$

If the value $I_{(i)} = 1\, kg \cdot m^2$ is accepted than the ratio $I_{(i)}/I_{(f)}$ will be displayed in the following row:

$R_{(f)}$	$0.5\,R_{(i)}$	$0.4\,R_{(i)}$	$0.3\,R_{(i)}$	$0.2\,R_{(i)}$
$I_{(i)}/I_{(f)}$	16	39.06	123.46	625

The given number of numerical values showed extremely powerful changes of values of the moment of inertia of the roll.
The decrease in the moment of inertia of the roll indicated that as such a transformation occurred, there was a corresponding decrease in the dynamic components of the loads, which should be evaluated by a positive result. However, the reduction of the specified moment of inertia was accompanied by the same intense limitation of the run-out time at a constant run-out time of the driving mass or its programmed kinematic run-out - braking. Such an imbalance should be assessed by additional dynamic perturbation, which will result in destabilization of the tension of the film canvas.
Let's compare the changes related to the processes of running out at radius values $R_{(i)}$ i $R_{(f)}$.
Reducing the mass of the roll and, at the same time, its moment of inertia meant, other things being equal, a decrease in the level of kinetic energy at the beginning of the run. Let the displacement of the film canvas during the cycle time t_c to be a value ℓ_c. Then the average speed in movement is:

$$v_c = \ell_c / t_c \, . \tag{7}$$

The average angular velocity of the roll corresponds to this average value v_c:

$$\dot{\varphi}_c = v_c / R \tag{8}$$

Since the average velocity was stabilized, this means that the average angular velocity will be variable as the changes of the radius of the roll from $R_{(i)}$ to $R_{(f)}$. So at a radius $R_{(i)}$ the value will be smaller and, conversely, as R approaches the value $R_{(f)}$, the angular velocity $\dot{\varphi}_{(f)}$ increases. Then the kinetic energy of the system within the changes from $R_{(i)}$ to $R_{(f)}$ will be:

$$E_{(i)} = 0.5\ I_{(i)}\left(\frac{v_c}{R_{(i)}}\right)^2 ; \ E_{(f)} = 0.5\ I_{(f)}\left(\frac{v_c}{R_{(f)}}\right)^2 \tag{9}$$

Then the ratio of kinetic energies expended in the run-out mode to overcome the moments of friction forces in the supports of the roll holder is:

$$\frac{E_{(i)}}{E_{(f)}} = \frac{I_{(i)}}{I_{(f)}} \cdot \frac{R_{(f)}^2}{R_{(i)}^2} \tag{10}$$

But since it is shown that $\frac{I_{(i)}}{I_{(f)}} = \frac{R_{(i)}^4}{R_{(f)}^4}$, then $\frac{E_{(i)}}{E_{(f)}} = \frac{R_{(i)}^2}{R_{(f)}^2}$.

The equation which corresponds to the regime of the run-out mode of the roll-holder-roll system in the general case:

$$\ddot{\varphi} = -\frac{M_{res}}{I} \tag{11}$$

or for comparable conditions we have:

$$\ddot{\varphi}_{(i)} = -\frac{M_{res(i)}}{I_{(i)}} ; \quad \ddot{\varphi}_{(f)} = -\frac{M_{res(f)}}{I_{(f)}} \tag{12}$$

The change of resistance moments from the value $M_{res(i)}$ to $M_{res(f)}$ is associated with a decrease in the mass of the roll. In the first approximation, we assumed that these moments of resistance were proportional to the masses:

$$M_{res(i)} = km_{(i)}; \qquad M_{res(f)} = km_{(f)} \quad (13)$$

Then:

$$\ddot{\varphi}_{(i)} = -\frac{km_{(i)}}{I_{(i)}}; \qquad \ddot{\varphi}_{(f)} = -\frac{km_{(f)}}{I_{(f)}} \quad (14)$$

Hence the integration was obtained under the initial conditions $t_{(i)} = 0$, $\dot{x}_{(i)} = v_c$:

$$\dot{\varphi}_{(i)} = v_c - \frac{km_{(i)}}{I_{(i)}}t; \quad \dot{\varphi}_{(f)} = v_c - \frac{km_{(f)}}{I_{(f)}}t \quad (15)$$

Since the values $\dot{\varphi}_{(i)} = 0$ and $\dot{\varphi}_{(f)} = 0$ correspond to the end of the run-out, we determine the end time of the run:

$$t_{end(i)} = \frac{v_c I_{(i)}}{km_{(i)}}; \qquad t_{end(f)} = \frac{v_c I_{(f)}}{km_{(f)}} \quad (16)$$

Substitution of values $I_{(i)} = m_{(i)}\frac{R_{(i)}^2}{2}$ and $I_{(f)} = m_{(f)}\frac{R_{(f)}^2}{2}$ led to the form:

$$t_{end(i)} = \frac{v_c R_{(i)}^2}{2k}; \quad t_{end(f)} = \frac{v_c R_{(f)}^2}{2k} \qquad (17)$$

Then the ratio of run-out time at the beginning and end of the technological process is:

$$\frac{t_{end(i)}}{t_{end(f)}} = \frac{R_{(i)}^2}{R_{(f)}^2} \qquad (18)$$

However, the formation of packaging from rolled packaging material was accompanied by important compliance with minor changes in the tension of the film. In this regard, in any high-performance packaging system with periodic or continuous operation there was provided the function of depreciation and accumulation of film material.
The presence in the machines of the linear type of areas of packing, packaging, sealing, marking led to the need to take into account their features, stiffness, variable laws of motion of the driving masses, the presence of shock modes and loads.

3.2. Transients of interaction of driving and driven masses

The rules developed on the basis of the law of conservation of energy allowed to make transitions from physical systems to their models on the principles of equivalence of kinetic energies (reduction of masses), instantaneous powers (reduction of forces and moments of forces) and potential energies of deformations (reduction of rigidities) [18]. The presence of springy bonds in the form of film canvases, tapes, belts in the packaging lines means the expediency of using linear models. In Fig. 1 a two-mass scheme is shown, which corresponds to the case with the absence of technological or non-technological gaps. In this case, the springy bond with stiffness *c* can be loaded to the value of the resistance force P_{res} applied to the driven mass or unloaded. In the first case, the system startup is modeled in one stage, and in the second - in two stages.

In the one-stage case, the driving mass m_1 and the driven mass m_2 simultaneously begin to move from the moment of application of the driving force to the driving mass with the load of the springy connection in addition by the forces of inertia.

The load condition of the springy bond to the value of $P_{sp} = P_{res}$ in the systems of film movement was achieved through the use of compensation devices. Fulfillment or non-fulfillment of this condition, or the presence of a technological gap δ (Fig. 2) lead to a different number of stages of dynamic interaction of the masses. The sequence of stages is as follows.

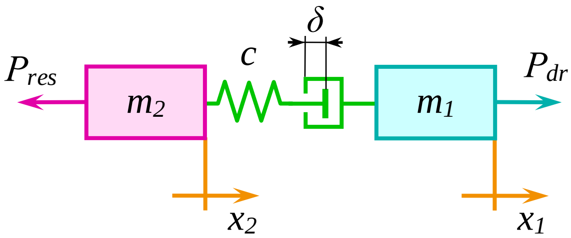

Fig. 2. Scheme of a two-mass model with a technological gap

The case of a system with a technological gap when $\delta \neq 0$ and $P_{sp} = 0$:

The overlap of the gap δ due to the movement of the leading mass m_1 under the action of P_{dr} corresponds to t*he first stage*:

$$m_1 \ddot{x}_1 = P_{dr}. \qquad (19)$$

As a result of integration of condition (1) we have the coordinate of movement x_1, the speed $\dot{x}_1$ of the driving mass with a sweep in time:

$$x_1 = \frac{P_{dr}}{m_1} \cdot \frac{t^2}{2}; \qquad \dot{x}_1 = \frac{P_{dr}}{m_1} t, \qquad (20)$$

final parameters of the first stage and time of its course:

$$x_{1(f)} = \delta; \quad x_{(f)1} = \frac{P_{dr}}{2m_1} t_{(f)}^2; \; t_{(f)}^I = \sqrt{\frac{2m_1 x_{1(f)}}{P_{dr}}}. \qquad (21)$$

The movement of the driving mass at the first stage had no counteraction, which under certain conditions may be accompanied by the achievement of a constant value of speed. In the case of using in the drives of the driving masses of motors with rigid characteristics, this can lead to shock loads at the next stage.

At the second stage, the springy element was loaded to the value of P_{res} by moving the driving mass and the stationary driven mass.

The equation of motion m_1 is written in the form:

$$m_1 \ddot{x}_1 = P_{dr} - c x_1, \qquad (22)$$

and the solution of the differential linear equation of the second order is carried out under the initial conditions:

$$t_{(i)}^{II} = 0; \quad x_{1(i)}^{II} = 0; \quad \dot{x}_{1(i)}^{II} = \dot{x}_{1(f)}^{I}. \quad (23)$$

Herewith

$$x_1 = A\sin\sqrt{\frac{c}{m_1}}\,t + B\cos\sqrt{\frac{c}{m_1}}\,t + \frac{P_{dr}}{c}, \quad (24)$$

where A and B are the integration constants that are determined after substituting the initial conditions (5). From here:

$$B = -\frac{P_{dr}}{c}; \quad A = \dot{x}_{1(f)}^{I}\sqrt{\frac{m_1}{c}}. \quad (25)$$

Then by substituting the constants of integration we finally get:

$$x_1 = \dot{x}_{1(f)}^{I}\sqrt{\frac{m_1}{c}}\sin\sqrt{\frac{c}{m_1}}\,t - \frac{P_{dr}}{c}\cos\sqrt{\frac{c}{m_1}}\,t + \frac{P_{dr}}{c}. \quad (26)$$

At the second stage, the springy force was determined by the dependence:

$$P_{sp} = cx_1 = \dot{x}_{1(f)}^{I}\sqrt{m_1 c}\sin\sqrt{\frac{c}{m_1}}\,t - P_{dr}\cos\sqrt{\frac{c}{m_1}}\,t + P_{dr}. \quad (27)$$

The substitution $P_{sp} = P_{res}$ corresponds to the completion of the second stage. Then:

$$P_{res} = \dot{x}_{1(f)}^{I}\sqrt{m_1 c}\sin\sqrt{\frac{c}{m_1}}\,t_{1(f)}^{II} - P_{dr}\cos\sqrt{\frac{c}{m_1}}\,t_{1(f)}^{II} + P_{dr}. \quad (28)$$

The value $t_{1(f)}^{II}$ from condition (28) was determined by iterations or using computer programs, according to which making the corresponding substitutions $x_{1(f)}^{II}$ and $\dot{x}_{1(f)}^{II}$ were defined as the initial conditions of the third stage. Regarding it, it is written:

$$m_1\ddot{x}_1 = P_{dr} - c(x_1 - x_2); \quad (29)$$

$$m_2\ddot{x}_2 = c(x_1 - x_2) - P_{res}. \quad (30)$$

The last two conditions are a general system of differential equations, which determines the motion of masses m_1 and m_2 at the given level. Transition from the real system to the given scheme of preliminary analysis with definition of driving and driven weights in packing machines by a link of reduction m_1 it is expedient to appoint a working body taking into account weights of the drive, and the driven weight m_2 selects a roll of packing material. In this case, the system has intermediate masses of rollers, the packaging material between the driving and driven masses, elements of the compensation mechanism and so on.

The combined actions of intermediate and main masses are estimated in the assumption that their effects are synchronous.

Obviously, for limited values of the stiffness of springy bonds, this assumption is not true. However, it meets the worst working conditions of springy connections, which should provide protection against overload.

Taking into account, that the intermediate masses have limited values compared to the driving and driven masses and the fact that the errors lead to increased loads and strength conditions, it is possible to come to the perception of such an assumption [19]. However, there is another borderline approach, at which the influence of intermediate masses is proposed to be neglected.

Formal rules for transforming the system of differential equations (29) - (30) allow to write the following:

$$\ddot{x}_1 + \frac{c}{m_1}(x_1 - x_2) = \frac{P_{dr}}{m_1}; \qquad (31)$$

$$\ddot{x}_2 - \frac{c}{m_2}(x_1 - x_2) = -\frac{P_{res}}{m_2}. \qquad (32)$$

Subtracting from condition (31) equation (32), we have:

$$(\ddot{x}_1 - \ddot{x}_2) + \left(\frac{c}{m_1} + \frac{c}{m_2}\right)(x_1 - x_2) = \frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}. \quad (33)$$

Since the task of the study was to determine the loads of the springy bond P_{sp}, it was advisable to make the transition to the equation of springy forces, multiplying all the components from conditions (33) by the given stiffness from the system:

$$c(\ddot{x}_1 - \ddot{x}_2) + c\left(\frac{c}{m_1} + \frac{c}{m_2}\right)(x_1 - x_2) = c\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right). \quad (34)$$

Since $c(x_1 - x_2) = P_{sp}$ and $c(\ddot{x}_1 - \ddot{x}_2) = \ddot{P}_{sp}$, we have:

$$\ddot{P}_{sp} + \left(\frac{c}{m_1} + \frac{c}{m_2}\right)P_{sp} = c\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right). \qquad (35)$$

The solution of the last condition is in the form:

$$P_{sp} = A_1 \sin\omega t + B_1 \cos\omega t + \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right), \quad (36)$$

$$\text{where } \omega = \sqrt{\frac{c}{m_1} + \frac{c}{m_2}}. \qquad (37)$$

The integration constants A_1 and B_1 are searched for under the initial conditions of the kinematic parameters:

$$t_{(i)} = 0; \quad x_{1(i)} = \frac{P_{res}}{c}; \quad \dot{x}_{1(i)} = \dot{x}^{II}_{1(f)}; \quad x_{2(i)} = 0; \quad \dot{x}_{2(i)} = 0. \qquad (38)$$

Then the initial conditions for the values of springy forces are:

$$P_{sp(i)} = c\left(x_{1(i)} - x_{2(i)}\right) = P_{res}; \qquad (39)$$

$$\dot{P}_{sp(i)} = c\left(\dot{x}_{1(i)} - \dot{x}_{2(i)}\right) = c\dot{x}^{II}_{1(f)}. \qquad (40)$$

Substitution of the initial condition $P_{sp(i)}$=PF_{res} leads to the value:

$$B_1 = P_{res} - \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right). \qquad (41)$$

To determine the integration constant A_1, the differentiation of condition (36) is performed:

$$\dot{P}_{sp} = A_1\omega\cos\omega t - B_1\omega\sin\omega t, \qquad (42)$$

From this it was obtained

$$c\dot{x}^{II}_{1(f)} = A_1\omega; \qquad A_1 = \frac{c\dot{x}^{II}_{1(f)}}{\omega}. \qquad (43)$$

Then we finally have:

$$P_{sp} = \frac{c\dot{x}_{1(f)}^{II}}{\omega}\sin\omega t + \left(P_{res} - \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right)\right)\cos\omega t + \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right). \quad (44)$$

From the latter condition it is seen that the total load of the springy bond is determined by the sum of static and two dynamic components. This means that the function $P_{sp} = P_{sp}(t)$ will have extrema. To determine them, it is necessary to find the value of the t_{ext} - the time of their achievement. After differentiation of the function (44) and equating the result to zero, we obtain:

$$t_{ext} = \left(arctg \frac{c\dot{x}_{1(f)}^{II}}{P_{res} - \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right)} \right)\omega^{-1}. \quad (45)$$

The found value of t_{ext} allowed to define extreme loading:

$$P_{sp\ ext} = \frac{c\dot{x}_{1(f)}^{II}}{\omega}\sin\omega t_{ext} + \left(P_{res} - \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right)\right)\cos\omega t_{ext} + \frac{c}{\omega^2}\left(\frac{P_{dr}}{m_1} + \frac{P_{res}}{m_2}\right). \quad (46)$$

It is obvious that the obtained value of $P_{sp.ext}$ allowed to determine the achievement of the condition of the strength of the canvas of springy bonding, for example, in the form:

$$\sigma_t = \frac{P_{sp.ext}}{F} \le [\sigma]_t, \quad (47)$$

which corresponds to the condition of tensile strength, where F is the cross-sectional area of the canvas; $[\sigma]_t$ is the allowable tensile strength of the material.

It is obvious that the factors influencing the achievement of the strength condition are the values of $P_{sp.ext}$ and F, which are variables. So if necessary there is a possibility in some step mode to choose the increased width or thickness of canvas or it is a possibility of decrease $P_{sp.ext}$. However, it should be taken into account that the increase in the cross-sectional area of the canvas requires a new recalculation of the value of the extreme load due to the increase in stiffness. The latter was determined by the properties of the canvas material and geometric parameters in the form:

$$c = \frac{EF}{\ell}, \quad H/m \quad (48)$$

where E – the modulus of springiness of the packaging material, Pa; ℓ – the length of the springy ligamenty, m.

The latter dependence followed from the formulation of the parameter c, because the stiffness was determined by the ratio of the force action on the sample to the amount of deformation caused by the action of this force. Since the force action was the value of $P_{sp.ext}$, and the value of the absolute deformation according to Hooke's law was:

$$\Delta\ell = \frac{P\ell}{EF}, \quad (49)$$

Then

$$c = \frac{P}{\Delta\ell} = \frac{EF}{\ell}. \quad (50)$$

In our case, the basis for the recalculation was an increase in the parameter F, and therefore it was necessary to re-determine $P_{sp.ext}$ and re-recalculate the strength condition. It was possible that obtaining the final result could require recalculation. In addition to the above reason, there was another caveat in such actions, which concerned the features of packaging lines in the modeling of the dynamics of force.

The model in equations (29) - (30) indicated the lack of consideration of the distributed mass of the flexible connection, which was due to one of the assumptions that the mass of the springy connection was limited and neglected.

At the same time, condition (44), although it provided an answer to the question of the value of springy loading, but it was almost impossible to assess the factors of influence and prospects for load limitation. A certain exception applied to the driving forces of P_{dr} and the forces of resistance P_{res}, because in the amplitude of the cosine component and in the component of static load they were written in numerators. This confirms the possible phenomenological conclusion about the increase in P_{sp} at their growth.

Analysis of the amplitude of the sinusoidal dynamic component indicated the importance of the influence of velocity $\dot{x}_{1(f)}^{II}$ and stiffness c, although the influence of this component was also veiled by the parameter ω, which reflected the natural circular frequency of oscillations of the dynamic system. Taking into account the fact that it was investigated equipment of cyclic action, the comparison of natural and external frequency of influences was relevant.

Evaluating the stiffness of polymer packaging films, the characteristics of polyethylene and polyethylene terephthalate materials as examples were taken into account.

Ranges of parameters, such as thickness and modulus of springiness of material, were accepted in calculations.

In the generalized estimate of the ranges of specific stiffness c, the thickness of the film materials was considered to be the root cause, without rejecting the effects of variable values of density, because in the classical sense, the modulus of springiness reflected the physical properties of materials. In addition, the length of the section ℓ was an important factor in destabilizing the stiffness values.

The significant influence of the length of the film section in the case of changes ℓ in the modes of interaction between the driving and driven mass indicated that the dynamic loads in the form of P_{sp} will be different. In addition, the natural oscillation frequencies of the system will be variable. In Figures 3 and 4 the graphical dependences c = c (ℓ) are shown, from which it follows that as the total value of ℓ decreases, the influences in values Δc increase.

This means that with changes in the values of Δc in response to changes in length ℓ dynamic models became nonlinear, with increasing stiffness the amplitudes of the dynamic components increased and there was an effect on the static component of the springy bond load. In addition, from the formula (44) follows the presence of the initial velocity of the third stage of the driving mass in the amplitude of the sinusoidal component. It was the final velocity at the previous stage $\dot{x}_{(f)}^{II}$, and therefore the worst situation in terms of springy bond loads corresponded to the shock interaction, when it was achieved $\dot{x}_{(f)}^{II} = \dot{x}_{1\max}^{II} = V$ during the selection of the gap δ. Recourse to such a case led to the condition:

$$P_{sp} = V\sqrt{m_2 c}\,\sin\sqrt{\frac{c}{m_1}}\,t + P_{res}, \qquad (51)$$

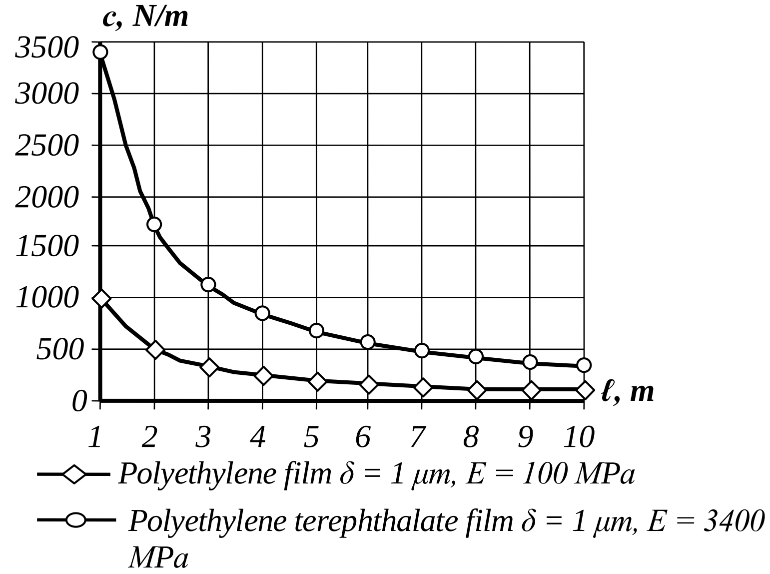

Fig. 3. The dependence of the stiffness of dynamic systems on the length of the flexible connection for limited values of film thickness

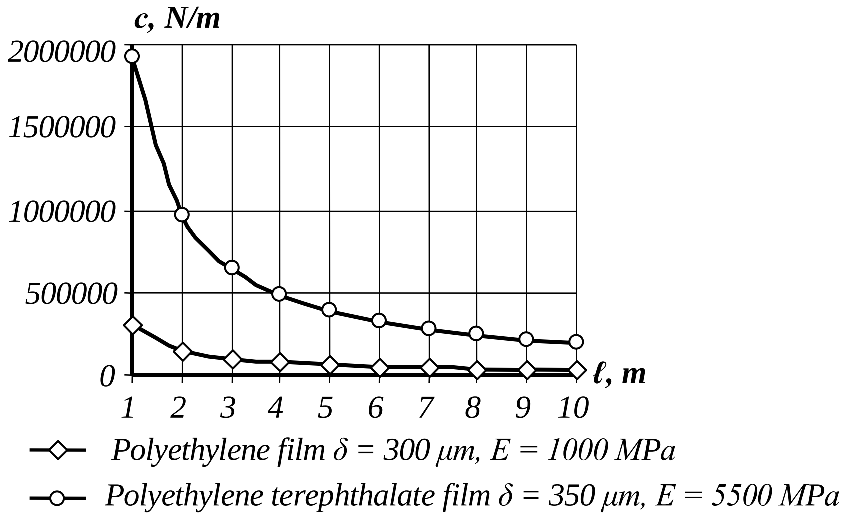

Fig. 4. The dependence of the stiffness of dynamic systems on the length of the flexible connection for larger values of film thickness

At this condition the springy load consisted of dynamic and static components. The maximum value of the dynamic component was reached at time t_m:

$$t_m = \frac{\pi}{2}\sqrt{\frac{m_2}{c}}, \tag{52}$$

Mathematical formalization (51) in contrast to the previously obtained equation (44) allowed to unambiguously assess the role of influencing factors. Thus, with respect to the amplitude of the dynamic component, the presence of three factors such as velocity V, driven mass m_2

and stiffness c of the elastic element was clearly defined. However, the structure of the amplitude of the dynamic sinusoidal component in model (44) formally coincided with (51), because we had:

$$A = \dot{x}_{(f)}^{II}\sqrt{\frac{cm_1 m_2}{m_1 + m_2}}, \qquad (53)$$

where complex $m_1m_2/(m_1+m_2)$ acted as a reduced mass of the system.

Comparing the two models, it should be concluded that the second model gave slightly more rigid results for P_{sp}, but calculations for them showed deviations in the range of 5 ... 8%, which in the descriptions of the dynamics of machine units was considered acceptable. However, from the point of view of the use of influence parameters in the interests of variations of P_{sp} values, there were certain limitations. Thus, the kinematic parameter of the velocity of the driving mass corresponded to a given system performance and its limitations were not always possible. The value of the driven mass was also technologically determined, and the reduction in stiffness could turn the system into a weakly damped, the disadvantage of which may have been violations in positioning and the need for dampers.

These three parameters of influence were dynamic factors, the change of which required additional analysis. This was all the more relevant due to the fact that they could be variable in technological use. For example, the fastening of the package-pallet with a stretchable film was accompanied by a variable speed of winding the film from the spool. The speeds changed periodically from minimum to maximum and again to minimum, which was accompanied by uneven tension of the film with the prospect of destruction of the package-pallets during transportation. In addition, the length of the film changed at limited values of this parameter and hence the variable stiffness in the hopping mode, followed by a nonlinear change. The driven mass of a roll with a film will be also variable, however these changes occur extended in time.

4. Conclusion

Modern theoretical basis for the synthesis of technological machines based on the use of film roll packaging materials combined the ability to take into account technological, economic requirements, high productivity, neighborhood conditions, energy savings, restrictions on dynamic loads and more. Achieving the combination of these requirements was largely due to the use of flexible connections directly between the working bodies of technological machines and in the packaging lines.

Synchronized combination of operations, the use of optimal laws of movement of working bodies, high-precision fabrication of elements of kinematic pairs and parts, limitation of dynamic loads, kinetic energy recovery measures were among the positive areas of creating efficient technological machines and lines of them.

Limiting the mutual influences on the performance of individual machines and lines in general due to intermediate compensators was a logical direction of synthesis of systems, but some caveats relate to the growth of material, economic and energy costs in the modes of their creation and operation.

Energy costs in the systems of movement of film, roll packaging materials are associated with the work of driving forces against friction and to create flows of kinetic energy of moving masses.

Unwinding of film rolls was accompanied by changes in their moments of inertia in

cycles from the beginning to the end of their use by two orders of magnitude.
The ratio of the run-out time of rolls of variable mass was determined by the ratio of the squares of the initial and final radii.
The presence in the lines of movement of film materials compensation and damping devices led to their division into almost two dynamically independent parts.
A variable nature of the moment of inertia of the roll with the film had significant destabilizing effect on the dynamics of the system. In this regard, it is advisable to create a compensator-regulator of the moment of inertia of the roll and the roll holder.
The solution of the problem of kinetic energy recovery in systems of periodic action is simultaneously connected with the restriction of the unevenness of the drives of machines.
In systems with a flexible bond of film material, the run-out mode of the driving mass is controlled, and the run-out of the driven mass can be controlled, provided that the acceleration by breaking it by module is greater than the acceleration module of the run-out of the driving mass. Otherwise, the springy connection between the working body and the driven mass is lost and there is a need to use a compensator-regulator.
Mathematical formalizations of interactions at the level of two-mass models in the stepwise mapping of kinematic and dynamic parameters, including with the use of equations of springy forces for the possibility of achieving strength conditions for flexible connections, have been developed.
A method for estimating the stiffness of springy bonds based on film materials taking into account their geometric parameters is proposed. It is shown that the variable transient value of the springy bond length acts as a dynamic exciter of the system.

5. References

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